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GRAVITATIONAL RADIATIVE FRICTIONAL FORCE

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D. D. Ivanenko showed qualitatively (Uspekhi Fiz Nauk 32, No. 2, 3, 1947) that, besides electromagnetic and meson friction, gravitational radiative friction must also exist, which must be taken into consideration in equations of motion of an individual particle. Developing these considerations we computed, in the framework of the classical theory, the retardation, caused by the radiation of gravitational waves, for a point particle in an approximation which can be called semirelativistic in a sense to be clarified below. Such calculations are of interest in the general theory of fields and particles.

The equation of the geodesic line in space warped by the natural gravitational field is regarded as the equation of motion of a particle. This field is considered to be weak, thus allowing Einstein's equations to be used in a linear approximation; the discussion is conducted in a geodesic normal system of coordinates with the origin coinciding with the particle's position at a certain moment. The applicability of these premises for realizable cases will be considered below.

Small additions  $h_{\alpha\beta}(x)$  to the Galilean values of the metric tensor are determined by the equation  $\square \psi_{\beta}^{\alpha} = -16\pi k t_{\beta}^{\alpha} / c^4$  (1) where  $h_{\beta}^{\alpha} = \psi_{\beta}^{\alpha} - \delta_{\beta}^{\alpha} \psi_{\gamma}^{\gamma}$ , with normed conditions of the Hilbert-Lorentz type:  $\delta \psi_{\alpha}^{\beta} / \delta x^{\alpha} = 0$  (summation is carried out with respect to identical indices).

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Here we assume  $t_{\beta}^{\alpha}$  equal to the energy tensor:  $t_{\beta}^{\alpha} = m_0 c^2 u^{\alpha} u_{\beta}$ .  
 $\cdot (1 - \frac{v^2}{c^2})^{\frac{1}{2}} \delta(x^{\gamma} - x^{\gamma}(t))$ , where  $x^{\gamma}(t)$  is the particle's  
 coordinate,  $m$  is the natural mass,  $k$  is the Newtonian gravitational constant,  $u^{\alpha} = dx^{\alpha}/ds$  and  $ds^2 = -dx^0^2 + dx^1^2 + dx^2^2 + dx^3^2 + h_{\alpha\beta} dx^{\alpha} dx^{\beta}$ .

As will be evident from a consideration of solution (3) utilized below, we cannot, thanks to the peculiarities of this problem, include, in our approximation, the squares of the derivatives of the gravitational potentials into the equations determining the spatial components  $\psi_{\beta}^{\alpha}$ . (See V. A. Fok, ZhETF Vol. 9, 375, 1939).

Instead of proceeding only from the field equations, we employ in the right side of equation (1) above the expression taken from the special theory of relativity. This represents an application of unique successive approximations, which turns out to be convenient, since we are conducting our discussion in a space differing slightly from a pseudoscalar one.

For the same reason we can generally speak of a weak gravitational field and also approximately introduce the notion of a vector in its ordinary sense by attaching to  $x^1, x^2, x^3$  the significance of spatial Cartesian coordinates, and to  $t = x^0/c$  the significance of time. The method of successive approximations was used earlier, by V. A. Fok, Infeld, Hoffman, P. Bergman, and N. M. Petrova (ZhETF, Vol. 19, 989, 1949), with the more general aim of obtaining the equations of motion of a Newtonian system from the equations of the gravitational field.

Birkhoff indicated the reasons for the selection, in Riemannian space of the general theory of relativity, of a geodesic normal system of coordinates with Galilean significance of the metric tensor  $g_{\alpha\beta}(x^{\gamma})$  for  $x^{\gamma} = 0$ , which (i.e. system), as we shall consider, approximates most of all the Lorentz system. As follows already from the geometric sense of geodesic coordinates, the approximation used by us will be valid if the particle does not leave

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during the time of observation the bounded region of three-dimensional space. The conditions that the additions  $h_{\alpha\beta}(x)$  to  $g_{\alpha\beta}(0)$  be small for  $x^\gamma \neq 0$  and, consequently, the conditions that the gravitational effects be small are described in the form of inequalities:  $R_{\alpha\beta} l^2 < 1$ , where  $R_{\alpha\beta}$  is Ricci's tensor and  $l$  is a linear measure of a part of the three-dimensional space under consideration. The inequality derived follows already from considerations of dimensionality; one can obtain it by considering the Maclaurin expansion of  $g_{\alpha\beta}$ . Since the greatest of the values of  $R_{\alpha\beta}$ , in the order of magnitude, equals  $kt_{\infty}/c^4 = kp/c^2$ , where  $p$  is the density of the mass, our condition can be rewritten thus:  $l < c/\sqrt{kp}$ .

For elementary particles, one has to replace, in a certain sense in the spirit of the field theory of matter,  $p$  by  $m/r_g^3$ , where  $r_g$  is the so-called gravitational radius. Characteristically, in the last case the necessity appears particularly clearly for a quantum-mechanical treatment in discussions of finite situations.

As is easily seen, any solution (1) of  $\psi_\beta^\alpha(x)$  can be transformed by an approximate expression of the form:  $-A_{\beta\gamma}^\alpha x^\gamma - \psi_\beta^\alpha(0)$ , where  $A_{\beta\gamma}^\alpha$  and  $\psi_\beta^\alpha(0)$  are constants in a given system of coordinates, so selected (i.e. constants) that  $\delta h_\beta^\alpha / \delta x^\gamma|_0 = 0$  and  $h_\beta^\alpha(0) = 0$ . Obviously these supplementary terms do not disrupt norming conditions. Thus we actually can conduct calculations in the normal system of coordinates and utilize the solutions of equations (1). In our succeeding computations, the terms with  $A_{\beta\gamma}^\alpha x^\gamma$  and  $\psi_\beta^\alpha(0)$  disappear, and therefore for brevity's sake they can henceforth be disregarded.

After simple transformations of the equation of the geodesic line:

$$\frac{\partial^2 x^i}{\partial t^2} = -\Gamma_{\alpha\beta}^i \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} + \frac{d^2 s/dt^2}{ds/dt} \frac{dx^i}{dt}$$

$$0 = -\Gamma_{\alpha\beta}^0 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} + c \frac{d^2 s/dt^2}{ds/dt}$$

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we obtain with the accuracy assumed the equation of motion:

$$m_0 \frac{d^2 F}{dt^2} = -m_0 c^2 \left\{ \frac{d\bar{h}_0}{cdt} - \frac{1}{2} \text{grad } h_{\infty} + \frac{\bar{v}}{2c^2} \frac{\partial h_{\infty}}{\partial t} + \frac{2v^i}{c} \left( \frac{d\bar{h}_i}{dt} - \frac{1}{2} \text{grad } h_{0i} + \right. \right. \\ \left. \left. + \frac{\bar{v}}{2c} \frac{\partial h_{00}}{\partial x^2} \right) + \frac{v^i v^j}{c^2} \left( \frac{\partial \bar{h}_{ij}}{\partial x^j} - \frac{1}{2} \text{grad } h_{ij} + \frac{\partial h_{0j}}{\partial x^j} \frac{\bar{v}}{c} - \frac{\partial h_{ij}}{\partial t} \frac{\bar{v}}{2c^2} \right) \right\}.$$

The Greek indices run through the values 0, 1, 2, 3 and the Latin run through 1, 2, 3. For abbreviation, we employ the symbolism of three-dimensional vector analysis and introduce the designations  $\bar{h}_\alpha = h_\alpha^i e_i$  where  $e_i$  are constants which can approximately be interpreted as three-dimensional bases.

Thus, as in a similar problem of electrodynamics we have now an equation (2) of motion and a wave equation (1) for the potentials of a natural field of a particle. Using the mentioned similarity, one can take, as in the corresponding computations of Lorentz, for substitution in (2) the solution of equations (1) in the form of lagging potentials (the feasibility of the norming condition was earlier pointed out by Einstein); then one can expand in a series in  $1/c$  and reject the divergent terms connected with the field gravitational mass.

The calculations are simplified somewhat by the fact that, as follows already from qualitative considerations, in the expression for the gravitational radiative frictional force one must introduce only the terms with:  $1/c^3, 1/c^5, \dots, 1/c^{2n+1}$ , where  $n$  is a whole number.

A similar finite result is obtained more simply by the method of Wentzel, Dirac, and A. A. Sokolov (see ZhETF, 18, 280, 1948), who utilized the half-difference of the leading and lagging electromagnetic potentials, which automatically excludes divergent terms in the series expansion in  $1/c$ .

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Using this method of electrodynamics for gravitation, we take:

$$\begin{aligned}
 h_{\beta}^{\alpha} &= \frac{2m_0 k}{c^2} \int \frac{u^{\alpha} u_{\beta} + \frac{1}{2} \delta_{\beta}^{\alpha} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}}{R(t')} \left\{ \delta(t' - t + \frac{R}{c}) - \delta(t' - t - \frac{R}{c}) \right\} dt' \\
 &= -\frac{2m_0 k}{c^3} \left\{ 2 \frac{\partial}{\partial t} \left[ (u^{\alpha} u_{\beta} + \frac{1}{2} \delta_{\beta}^{\alpha} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}) + \frac{1}{3 c^2} \frac{\partial^3}{\partial t^3} [R^2 (u^{\alpha} u_{\beta} + \frac{1}{2} \delta_{\beta}^{\alpha} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}) \right. \right. \right. \\
 &\quad \left. \left. \left. + \frac{1}{5! c^4} \frac{\partial^5}{\partial t^5} [R^4 (u^{\alpha} u_{\beta} + \frac{1}{2} \delta_{\beta}^{\alpha} (1 - \frac{v^2}{c^2})^{\frac{1}{2}}) \right] + \dots \right\} \quad (3)
 \end{aligned}$$

Here  $R = |\vec{r} - \vec{r}_1(t)|$ , where  $\vec{r}_1(t)$  is the running radius-vector of the particle. After substituting  $h_{\beta}^{\alpha}$  into (2), where we can let  $R = 0$  and  $\partial \vec{r} / \partial t = -\vec{v}(t)$ , we obtain the desired equation:

$$m_0 \frac{d\vec{v}}{dt} = -11 \frac{m_0^2 k}{c^3} \left[ \frac{1}{3} \frac{d^2 \vec{v}}{dt^2} + \frac{1}{6} \frac{v^2}{c^2} \frac{d^2 \vec{v}}{dt^2} + \frac{1}{c^2} (\vec{v} \frac{d\vec{v}}{dt}) \frac{d\vec{v}}{dt} + \dots \right]$$

Here the terms containing  $1/c^7$  or higher powers of  $1/c$  are not written, which causes us to call the computation semirelativistic.

If the particle moves without acceleration, then the expression obtained after averaging with respect to time will not lead to the radiation of energy since  $\vec{v} d\vec{p}/dt$  reduces to a complete derivative; for example,  $v^2 \frac{d^2 \vec{v}}{dt^2} = \frac{1}{2} \frac{d^2 v^2}{dt^2} + (\frac{d\vec{v}}{dt})^2$ . The appearance in the expression, for the radiation of energy of a point particle, of a term with  $1/c^3$  for  $d\vec{v}/dt \neq 0$  does not contradict the well-known formula for the radiation of gravitational energy by a system in which only a quadrupole term proportional to  $1/c^5$  is contained. As in the case of radiation of electromagnetic energy by a moving electron, the origin, during acceleration, of the dipole term proportional to  $1/c^3$  is connected with the fact that the expansion into dipole, quadrupole, etc terms is not invariant. However, it is necessary to note the essential fact: if one integrates at once the equation of motion of a point-particle with retarding force, then solutions with  $d\vec{v}/dt \neq 0$  will appear. In other words, the fact of the presence of a natural gravitational field poses the question: whether it is possible in principle to choose,

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within the framework of the theory of relativity, a system of readings and measurements in which the acceleration of a body would differ from zero for a velocity  $\vec{v} \neq 0$ . The situation here is completely analogous to the problem of Dirac's self-accelerating electron in the classical theory.

In conclusion we note that our approach to the problems of gravitation, based (i.e. approach) on the equations of a weak field, permits one to establish still another series of problems, for example the problem of gravitational vacuum. On the other hand, the general nonlinear theory of the gravitational field must also lead to a suitable radiative frictional force.

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